improved to allow investigation of more complex planforms such as planforms with streamwise side edges. Use of the conservative estimate of $C_B$ and load distribution calculated using the suction analogy may be justifiable in applications where the additional structural weight penalty is not severe.


## Acknowledgment

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# Test of Pines' Approximate Method in a Flutter Calculation for the Viggen Aircraft


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PINES[1] and Landahl[2] emphasized long ago that flutter is often associated with a loss of resultant stiffness. Using this observation as a basis, Pines[1] proposed an approximate method for calculation of the flutter speed. This method was tested in a large number of cases by Ferman[3], and Pines and Newman[4] concluded that the method was "...quite accurate for primary surfaces and reasonably acceptable for control surfaces." By this Note we wish to show that the method seems to be useful also for a canard configuration such as that of the Viggen aircraft.

The approximate method has been applied to the Viggen configuration and the result is compared here to that of a corresponding flutter calculation by the $p$-method. In both calculations, a linear combination of five symmetric natural modes plus the rigid translation and pitch modes was used. The deflection in the natural modes mainly consists of wing bending, body bending, engine motion, wing torsion, and wing motion in the wing plane. These modes, the mass matrix, and the stiffness matrix were determined by means of data from a ground vibration test, and the aerodynamic matrix was calculated by the Polar Coordinate Method[5] for some values of the reduced frequency $k$. An approximation to the aerodynamic matrix then was determined in the form of a polynomial in $ik$ with real coefficient matrices. This was used in the flutter calculation by the $p$-method, whereas only the zero-order coefficient matrix was used in the calculation by the approximate method.

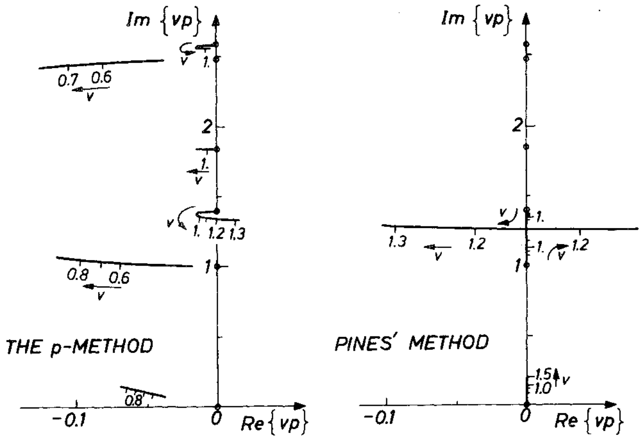


Fig. 1 Root loci in the complex $vp$ plane for a Mach number of 0.7.

Let $U$ denote the speed of flight, $L$ a reference length, $\omega_n$ the frequency of the $n$th natural mode, and $p=\mu+ik$ the dimensionless complex frequency parameter that appears upon Laplace transformation of the equations of motion with respect to the dimensionless time $\tau=Ut/L$. The eigenvalues $p=p_n$, which are roots of the characteristic equation and vary with the normalized speed $v=U/(\omega_l L)$, form loci in the complex $p$-plane. We prefer, however, to consider the corresponding values of the product $vp=(U/\omega_l L)(p'L/U)=p'/\omega_l$ and the loci of these in the $vp$ plane. The values of $vp$ for $v=0$ represent starting-points for the loci. If the virtual masses are small, the moduli of the starting points which lie on the imaginary axis are approximately equal to the frequency ratios $\omega_n/\omega_l$.

The results of the calculations by the $p$-method and the approximate method are illustrated by the two diagrams in Fig. 1. It is seen that it is the wing bending mode and the body bending mode which couple and form the aeroelastic mode that goes unstable. For an increasing subcritical speed, the roots obtained by the approximate method for these two modes lie on the imaginary axis and move toward each other until they meet. This occurs at a speed which, according to Pines, may be considered an approximation to the flutter speed. For still higher speeds, the roots leave the imaginary axis in opposite directions.

The corresponding loci in the diagram obtained by the $p$-method are to some extent similar. However, those parts of the loci which correspond to subcritical speeds lie to the left of the imaginary axis and the roots do not reach each other. They move also in this case essentially in opposite directions away from each other as soon as a speed slightly less than the flutter speed has been exceeded.

From the loci produced by the $p$-method and the approximate method we may read the values 1.20 and 1.15, respectively, for the flutter speed. The latter is only 4% lower than the former. We therefore conclude that the approximate method is useful also for a canard configuration.

We may add that a satisfactory flutter margin exists. The flutter speed result $v=1.2$, which applies to standard day sea-level density and zero angle of incidence, implies that the flutter speed at a flight Mach number of 0.7 is twice as large as the corresponding flight speed at standard day sea-level temperature.


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# Fracture Mechanics Correlation for Tensile Failureof Filamentary Composites with Holes


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## Introduction

HOLES in filamentary composite materials are of great and continuing interest, since mechanical fasteners are important for joining components together. Holes in ductile metals do not affect the static tensile strength, because the plasticity of the stress-strain behavior smooths out the stress distribution at failure. Since such filaments as boron and graphite are inherently brittle, almost all new designs must first determine the failure characteristics of the laminate layup with holes.

Waddoups et al.[1] presented results indicating that filamentary composite materials with holes exhibited a behavior similar to that exhibited by ductile metals with cracks. However, in order to use linear elastic fracture mechanics, they postulated the existence of intense energy regions at the edges of the hole in which the fracture process takes place. These intense energy regions are modeled as edge cracks emanating from a circular hole, and then linear elastic fracture mechanics is applied. With this model, Waddoups et al.,[1] by using the Bowie solution for stress intensity, defined a fracture toughness $K_Q$ and demonstrated a correlation with the hole size.

Whitney et al.[2] have formulated a stress fracture criterion that assumed that failure occurs when the stress at a distance $d_0$ ahead of the hole reaches the tensile strength $\sigma_0$ of the virgin material (no hole). They also have an alternate criterion that assumes that failure occurs when the average stress over a distance $a_0$ ahead of the hole reaches the virgin strength. The effect of hole size is encompassed in these criteria, because the stress distribution in the vicinity of the hole explicitly contains the size of the hole.

All of the above theories correlate reasonably well with experimental data and appear to be adequate for purposes of engineering design, given sufficient experimental data. The above theories implicitly assume the material to be homogeneous and then apply linear elasticity to the devleopment of what might be called a "macroscopic" theory of failure.

This Note proposes a "microscopic" theory, which postulates that fracture of a filamentary composite initiates at a crack lying in the matrix material at the interface of the matrix/filament (thus explicitly recognizing the filament and matrix as two different materials). The effect of hole size is then shown to correlate with the singularity at the tip of this crack, which is at the interface of two different materials.

## Theory

The motivation for this theory comes from linear elastic fracture mechanics as applied to homogeneous materials where the fracture stress $\sigma_f$ in the presence of a crack is given by

$$\sigma_f \propto K_{IC}(2a)^{\frac{1}{2}} \tag{1}$$

Thus, $K_{IC}$ is a material parameter called the fracture toughness, which is experimentally determined and which has dimensions of stress × length to the one-half power. The crack size effect ($2a$ is the length of the crack) is embodied in the exponent of $-\frac{1}{2}$, which is the order of the mathematical singularity at the tip of the crack.

The present theory proposes an equation for the fracture stress $\sigma_f$ of filamentary composites of the form

$$\sigma_f = H(2a)^{-m} \tag{2}$$

where $H$ is akin to $K_{IC}$ but has dimensions of "stress × length to the $m$ power," which is different than that for $K_{IC}$. The exponent $-m$ is the order of the singularity of a crack with its tip at the interface of two different materials. It has been shown that the order of the singularity is a function of the ratio $\mu_1/\mu_2$ of the shear moduli of the two materials and of the two Poisson ratios $\nu_1$ and $\nu_2$.[3,4] Table 1 gives values of this singularity for a selected range of parameters.

Not much statistical data on the shear moduli of boron filaments, graphite filaments, and the epoxy resins are available. If the shear moduli of the boron and 6061 aluminum are taken as $24 \times 10^6$ and $3.8 \times 10^6$ psi, respectively, then the ratio of moduli for the boron/aluminum system is approximately 0.16. If the graphite filament and the epoxy resin tensile moduli are taken as $28 \times 10^6$ and $0.7 \times 10^6$ psi, respectively, then the ratio is approximately 0.025.

## Experimental Justification

In Fig. 1, fracture data from several different experiments are shown plotted in the format of $\log\sigma_f/\sigma_0$ versus $\log 2a$. It is be to observed that two different material systems are shown. One system, consisting of experimental data[8] obtained during the course of this investigation, is the 5.6-mil-diam boron

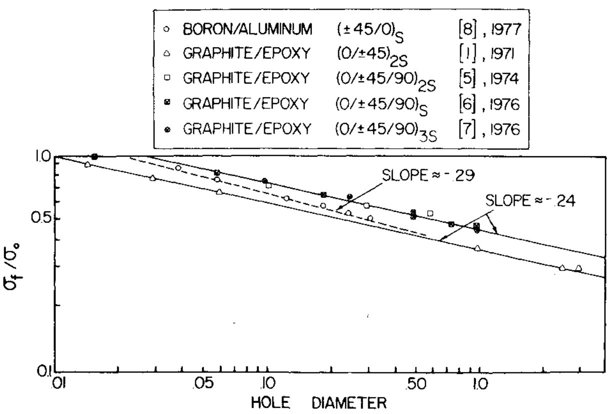


**Fig. 1 Experimental data.**


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